Calibrated linear methods for analysis and design of yielding RC structures

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ABSTRACT

This paper explores the development of a method that is useful for design of reinforced concrete (RC) frame structures to resist earthquakes. The method, originally proposed in the 1970s, makes an analogy between viscously damped linear and hysteretic response for the purpose of estimating maximum displacement. Recent dynamic test results are used to extend significantly the calibration of the method.

DRIFT AS A DESIGN CONSIDERATION

Despite the terminology and packaging associated with regular practice in designing for earthquake resistance, what really matters is how a structure performs if it is shaken by a significant earthquake. Figure 1 represents load-displacement responses of two structural systems. Conventional structural design philosophy (Fig. 1a) is understood in terms of demand and supply along the load axis. Some time ago, designers came to a realization that there were several aspects of the earthquake problem (dynamic equilibrium, loading that is unpredictable in terms of amplitude, frequency content, duration, and probability of occurrence) that made it implausible to fit into this conventional outlook. When structural response is idealized as linear (Fig. 1a), the relationship between displacement and load demands are clear, so displacements are of secondary importance and the attention they are given amounts to a verification. When response is nonlinear (Fig. 1b), the displacement axis is the only meaningful indicator of demand. Load demand loses physical meaning once it exceeds the supplied resistance.

All performance-related questions in design for earthquake resistance can be related to displacement demand:

1. Will story drift be too much for attached elements?
2. Will displacement cause collision with adjacent structures?
3. Will drift level demand too much deformability from the elements?
4. Will displacements bring about excessive secondary effects?

If it could be shown that a structure satisfied the above questions, then its supply of resistance along the load axis would be inconsequential.

Many current codes give the impression that design can be managed by control along the load axis in a fashion similar to wind design. Load demand is determined and then reduced by a quantity that is sensitive in an empirical way to the redundancy and deformability inherent to various structural systems. The resistance to be provided is linked to this reduced design load and it is understood that this implies yielding response in the design earthquake. Displacements are calculated for the purpose of checking only after design loads have been set. Displacement response is assumed to be linearly related to the unreduced load demand. Because of the strong parallels to linear design (as say for wind), the framework of this approach conceals the fundamental differences in demand

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for yielding systems and concentrates the designer's efforts to evaluate performance into selection of a few empirical constants. While the parallel with design for other kinds of lateral loads can be convenient, the code approach has the effect of calling attention to the less important axis of response.

Because displacements are directly related to questions about performance, the ability to base earthquake design requirements on realistic displacement demand is essential. This paper reviews a class of methods useful for estimating response of yielding reinforced concrete systems to earthquakes. Results of recent dynamic tests are used to motivate an extensive second look at a comprehensive method for design of RC frame structures that enables the designer to control damage on the basis of a realistic estimate of nonlinear displacement response.

LINEAR EQUIVALENTS OF YIELDING SYSTEMS

Putting aside the uncertainties associated with detailed prediction of earthquake characteristics, the task of computing dynamic response becomes tedious if the subject structure is likely to yield. When this happens, a considerable amount of energy can be dissipated through inelastic action of elements. Algorithms of varied sophistication have been developed to describe hysteresis in reinforced concrete with a certain degree of reliability. But such all-out nonlinear dynamic structural analyses are seldom appropriate for design because they tend to be too specific and out of proportion with the overall design effort.

Several alternative approaches for determining earthquake response have made use of response spectra, which are more intuitive and familiar tools. While it is possible to generate nonlinear response spectra using a selected hysteresis model, these are much more abstract than linear spectra, which form the basis for most design codes. That linear response spectra can be made to furnish values matching the response of a yielding system requires only a superficial argument. Consider a yielding system with initial period T and viscous damping ratio β for which it is known that the maximum response is D_{max}. The hypothetical case depicted in Fig. 2 shows four ways that a linear spectrum can be manipulated to furnish D_{max} by altering the nominal period or damping or both. While the argument is attractive, its feasibility would depend on the development of rules for selecting altered period and damping so that reliable results can be obtained.

The striking analogy that exists between viscous linear and hysteretic systems has been investigated extensively. The object of this section is to trace the development of concepts related to linear analogies of yielding systems. Particular emphasis is given to applications to reinforced concrete as a prelude to the next section, which is an extensive second look at the foundations of the substitute structure method.

Jacobsen (1930) introduced the notion of equivalent viscous damping to approximate the effects of friction proportional to general powers of velocity. The approach was used to estimate response amplitudes for single-degree-of-freedom (SDOF) systems with linear spring stiffness to sinusoidal loads.

Soon after Housner (1956) argued the inevitability and possible benefits of inelastic response of structures in earthquakes, the first suggestions were made to extend the idea of equivalent viscous damping to hysteretic systems (Jacobsen 1960, Ando 1960). Jennings (1964) summarized and compared several ensuing approaches for representing elasto-plastic SDOF oscillators as contrived linear systems. By assigning prescribed values of mass, stiffness and damping ratio, matching of resonant amplitude (and, in some cases, frequency) was achieved for steady-state response to sinusoidal loading. The closed-form nature of the expressions for substitute properties could not be maintained for earthquake problems because the more random exciting force prevented development of steady-state response, which is an essential condition to establishing equivalence in energy dissipation.

Takeda et al (1970) performed static and dynamic tests of SDOF reinforced concrete elements that led to the development of the first hysteresis algorithm tailored specifically to reinforced concrete. The model used a trilinear primary curve and provided expressions for unloading slope as a function of element ductility ratio and previous response history.

By viewing equivalent viscous damping in average fashion rather than on a cycle-by-cycle basis, Gulkan and Sozen (1974) did away with the restriction of steady-state response. Rather than deriving expressions mathematically, values for substitute frequency and damping were deduced from results of a series of dynamic tests of SDOF RC bents. Substitute frequency was taken as the ratio of measured maximum absolute acceleration to measured maximum displacement response, which is related to the apparent stiffness that would be observed in load-displacement relationships. Substitute damping for the yielding RC frames was computed on the assumption that the energy intake of the system over the duration of motion was balanced by a linear dashpot in order for the system to come to rest. This amounted to determination of the appropriate value of β_s in the expression

$$\beta_s\ [2m\omega_s \int_0^{t_f} \dot{x}^2 dt] = -m \int_0^{t_f} \ddot{y}\dot{x}dt \tag{1}$$

where β_s was the substitute damping ratio, m the mass, ω_s the substitute circular frequency, $\dot{x}$ the relative velocity response, $\ddot{y}$ the base acceleration, and t_f the duration of shaking. It was shown that measured displacement response was suitably approximated from linear spectra of the actual base motion (14 runs sinusoidal, 4 runs simulating El Centro NS 1940, and 10 runs simulating Taft N21E 1952) when the substitute properties (ω_s and β_s) were used. This result was then formulated into a method for determining design base shear based on an admissible ductility ratio, μ. For this purpose, it was suggested to lengthen the period based on cracked sections by $\sqrt{\mu}$ for substitute period (replacing ω_s). Substitute damping was related to μ with an expression patterned after the Takeda (1970) hysteresis model

$$\beta_s = 0.02 + 0.2\ (1 - 1/\sqrt{\mu}) \tag{2}$$

which was shown to provide a reasonable representation of substitute damping ratios deduced from test measurements.

The substitute structure method (Shibata and Sozen 1976) is the extension of Gulkan's SDOF base shear prescription to RC frames with more than one degree of freedom. Substitute period and damping were based on tolerable damage (ductility) ratios, μ_i, for various elements of the frame. In this manner, the designer could establish both the extent and relative distribution of damage to beams and columns. For each element, stiffness was reduced by $1/\mu_i$ times the value for cracked section. Substitute frequency was computed from linear analysis using reduced stiffness values. To compute substitute damping ratios for the full frame for individual modes, substitute damping ratios computed for each element on the basis of μ_i (Eq. 2) were weighted according to relative flexural strain energy associated with the mode shape:

$$\beta_m = \sum_i \left(\frac{P_i}{\sum_i P_i} \right) \beta_{si} \tag{3}$$

where β_m is the smeared modal substitute damping ratio for mode m, P_i is flexural strain energy for an element i in mode m, and β_{si} is the substitute damping ratio for an element i. With substitute properties thus defined, modal responses were read from linear spectra and combined to determine displacements, base shear, and member design forces. The principal benefit of the method was that it based design on issues related to performance (tolerable damage and drift) while requirements could still be stated in terms of design forces, as is customary for most other kinds of loading.

TESTS OF SDOF SYSTEMS

A critical step in applying the concept of equivalent viscous damping to earthquake problems was the shift toward reliance on dynamic test results as a basis for inferred damping. The foundations for the particular method considered in this study are based directly on 28 test runs by Gulkan (1974), for which only half used simulated earthquake motion, and indirectly on 7 test runs by Takeda (1970), for which only 3 used earthquake-like motions.

A recent series of dynamic tests (Bonacci 1989) has provided an extensive set of new data that is ideal for further investigation of linear analogies to yielding RC systems. The study provides 35 new test results for which simulated earthquake motions were used. The results are considered ideal not only because they triple the size of the data set, but also because they reflect the influence of four variables germane to nonlinear dynamic response: initial period and strength of the structure, and frequency content and intensity of the earthquake.

The specimen (Fig. 3) for this parametric study of nonlinear displacement response can be idealized as an inverted pendulum restrained by a flexural spring. The stocky panel, pinned at its base, formed the shaft of the pendulum, and steel plates mounted near the top accounted for most of the mass. The beam extending horizontally from the panel to a roller support (steel pipe column pinned at both ends) functioned as the spring for the pendulum. Detailing of sections was such that beam flexural strength was the weakest link in overall specimen resistance to lateral loads. Specimen initial period was controlled by varying panel height, beam span, and the amount of attached weight. Lateral-load strength was influenced mainly by the ratio of beam reinforcement provided, but also by the spans of the panel and beam. Base motions were patterned after three different earthquake records (Castaic N21E 1971, El Centro NS 1940, and Santa Barbara S48E 1952) to cover a wide range of frequency content. The ordinates of these records were scaled uniformly to vary intensity. A summary of the attributes for each of the test runs is given in Table 1.

Reduction of test results for the purpose of developing a linear response analog followed the procedures used by Gulkan (1974), with a few exceptions. Inspection of Eq. 1 reveals that calculation of substitute damping ratios requires a relative velocity response signal, $\dot{x}(t)$. Because no direct velocity measurements were made and because digital differentiation of recorded displacement response, $x(t)$, would be inherently troublesome, the required signal was computed as the time integral of available acceleration signals

$$\dot{x}(t) = \int \{[\ddot{x} + \ddot{y}](t) - \ddot{y}(t)\}\, dt \tag{4}$$

where $[\ddot{x} + \ddot{y}](t)$ is the absolute acceleration response at the center of mass and $\ddot{y}(t)$ is the base acceleration. The integral was very sensitive to baseline error in the parent signals. Gulkan (1974) applied a parabolic adjustment to the acceleration baseline. In the present study, digital filtering of extreme low-frequency components produced a satisfactory velocity signal. The process, as illustrated in Fig. 4, was to integrate the uncorrected relative acceleration signal ($\ddot{x}$; RHS of Eq. 4); compute the Fourier transform of the uncorrected velocity; filter components with frequency less than 1.25 Hz (the lowest response frequency deduced from zero-crossing rates of all 35 runs); transform back to the time domain. The resulting corrected velocity signals were checked by applying the same process for a second integration in order to compare with measured relative displacement response, $x(t)$. The only noted deviation was the elimination of displacement baseline offset as a result of filtering. Substitute damping ratios computed from 35 test runs (Table 1) ranged from 3 to 20% of critical.

Substitute frequency was computed from apparent stiffness (slope of a line joining unloading points in opposite quadrants) deduced from moment-rotation response of the pendulum in the cycle of peak rotation:

$$\omega_s = \frac{1}{r}\sqrt{k_a/m} \tag{5}$$

where r is the pendulum radius to the center of mass, m, and k_a is apparent stiffness.

Using these substitute properties (Table 1), an estimate of maximum displacement response (S_d) was made for each test run from a linear spectrum for the recorded base motion. These are compared with measured nonlinear response in Table 1 and Fig. 5, which illustrate that the linear analogy provided accurate estimates of peak response.

Substitute damping ratios are plotted against peak rotation ductility ratios (Table 1), μ, inferred from moment-rotation relationships in Fig. 6 along with the relationship (Eq. 2) proposed by Gulkan (1974). Substitute damping values computed from results of the 35 more recent tests were generally larger than those given by Eq. 2 at all values of ductility ratio. It follows that a revised best-fit equation could be obtained by increasing either or both of the constant (0.02) or linear multiplier (0.20) in the equation, but not by changing the order of the radical ($1/\sqrt{\mu}$). However, no modification to Eq. 2 is proposed for two reasons: (1) if only the current data were available to construct a design equation, a slight underestimate of damping would be judiciously conservative, and (2) many of the results from Gulkan's tests fell below the line given by Eq. 2.

CONCLUDING REMARKS

This paper traced the evolution of methods for representing yielding dynamic systems with equivalent linear analogs. Obstacles to applying such methods for earthquake problems in RC were overcome by placing some reliance on experimental results from a relatively small number of dynamic tests. This study considered results from more recent dynamic tests to triple the size of the relevant data set. It was shown that the approach proposed by Gulkan and Sozen (1974) provided an accurate organization of the more recent test results (Bonacci 1989) without need for modification. In itself, this is an obscure result. But its real value is as a "vote of confidence" for the substitute structure method.

Any design routine should be sensitive to the scale that will be used to judge its effectiveness. For building structures in earthquakes, performance questions (such as those listed earlier) are clearly related to displacement response. Most recognized design approaches (building codes, capacity design philosophy) lack sincere consideration of the level of drift response. The method reinvestigated in this paper offers a workable alternative for performance-based design of RC frames in earthquakes.

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Table 1. Summary of SDOF experiments (Bonacci 1989).

Test	T_i (1) sec	C (2)	Run	Record (3)	Max $\ddot{y}$ g	μ	ω_s rad/sec	β_s	S_d (4) in	D_{max} in
B-01	0.086	1.34	1	ELC	0.97	1.4	33.5	0.052	0.69	0.58
			2	ELC	1.35	2.5	27.3	0.086	1.26	1.08
			3	ELC	1.54	3.1	24.3	0.106	1.42	1.35
B-02	0.13	0.39	1	ELC	0.78	3.0	15.5	0.127	1.33	1.08
			2	ELC	1.37	5.8	9.9	0.174	2.09	2.12
B-04	0.17	0.33	1	ELC	0.77	3.6	12.9	0.121	1.69	1.40
			2	ELC	1.33	5.4	9.8	0.151	2.30	2.08
B-05	0.14	0.79	1	ELC	0.79	2.0	19.9	0.068	1.59	1.10
			2	ELC	1.34	3.4	14.9	0.133	2.34	1.88
B-06	0.14	0.39	1	CAS	0.49	1.0	21.2	0.062	0.35	0.39
			2	CAS	0.93	2.5	16.0	0.077	0.90	0.90
			3	CAS	1.69	4.9	12.8	0.130	1.38	1.75
B-07	0.17	0.67	1	SAB	0.51	1.2	20.7	0.039	0.90	0.72
			2	SAB	0.70	2.3	16.0	0.105	1.79	1.50
B-08	0.16	0.33	1	SAB	0.30	1.3	17.7	0.052	0.75	0.59
			2	SAB	0.47	3.9	11.9	0.144	1.64	1.73
B-09	0.14	0.39	1	SAB	0.37	1.3	19.3	0.061	0.68	0.49
			2	SAB	0.66	5.5	11.4	0.189	2.12	2.14
B-10	0.17	0.67	1	ELC	0.56	1.6	18.4	0.059	1.18	1.10
			2	ELC	0.95	2.4	14.9	0.104	1.84	1.68
			4	ELC	0.95	2.5	14.9	0.105	1.84	1.74
B-11	0.087	0.67	2	CAS	0.88	2.3	22.7	0.098	0.68	0.71
			3	CAS	1.51	2.9	19.4	0.118	0.99	0.98
			4	ELC	1.51	6.1	14.0	0.199	1.82	2.03
B-12	0.089	1.34	1	CAS	0.66	1.0	34.4	0.037	0.58	0.49
			2	CAS	1.05	1.4	30.8	0.050	0.66	0.63
			3	CAS	1.45	1.8	27.7	0.080	1.08	0.85
B-13	0.091	1.34	1	SAB	0.55	0.8	34.8	0.032	0.37	0.42
			2	SAB	0.88	1.3	31.5	0.044	0.64	0.70
B-14	0.089	0.67	1	SAB	0.40	0.9	32.5	0.044	0.24	0.25
			2	SAB	0.77	2.5	22.7	0.096	0.82	0.85
			3	SAB	0.91	3.5	18.5	0.125	1.30	0.96
B-15	0.14	0.79	1	SAB	0.35	0.8	23.9	0.038	0.60	0.51
			2	SAB	0.78	1.7	19.2	0.067	1.13	1.14
			3	SAB	0.88	2.1	16.7	0.103	1.87	1.46

Notes:

(1) T_i = specimen initial period.
(2) C = specimen lateral strength coefficient = load at yield/weight.
(3) CAS denotes Castaic N21E 1971 motion.
ELC denotes El Centro NS 1940 motion.
SAB denotes Santa Barbara S48E 1952 motion.
(4) S_d = displacement from linear spectrum using substitute properties.

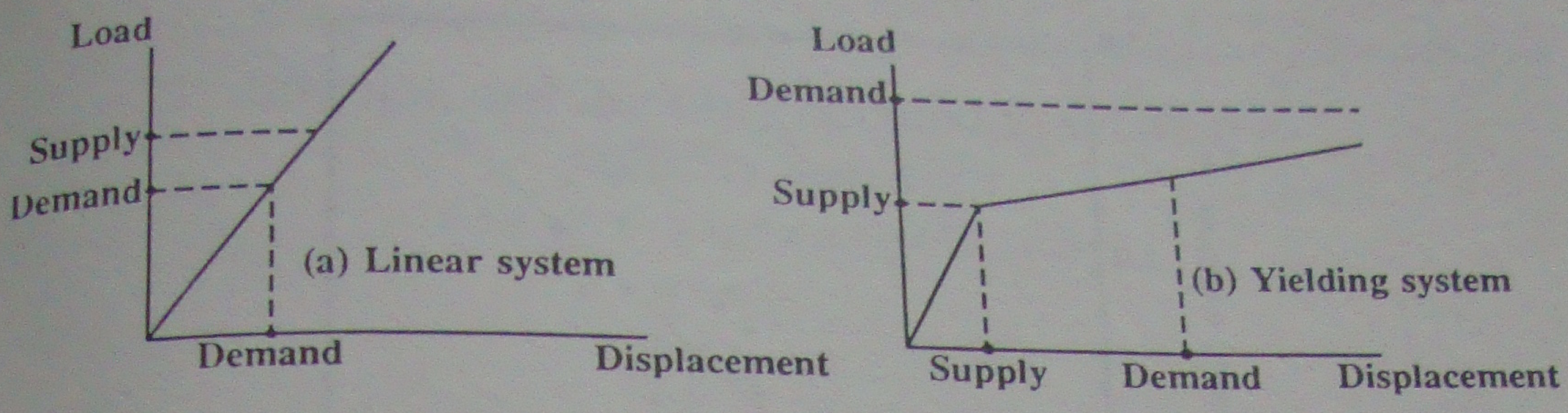

Figure 1. Design philosophy and structural response

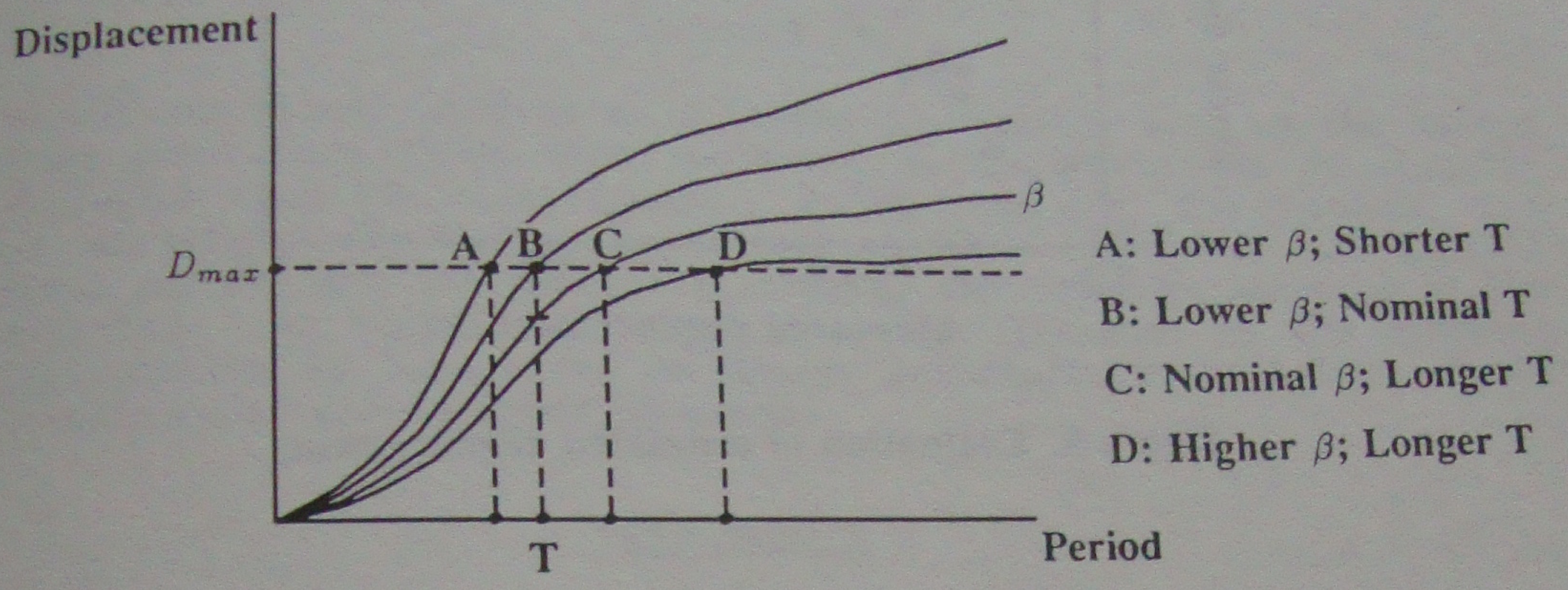

Figure 2. Altered properties for linear response spectrum

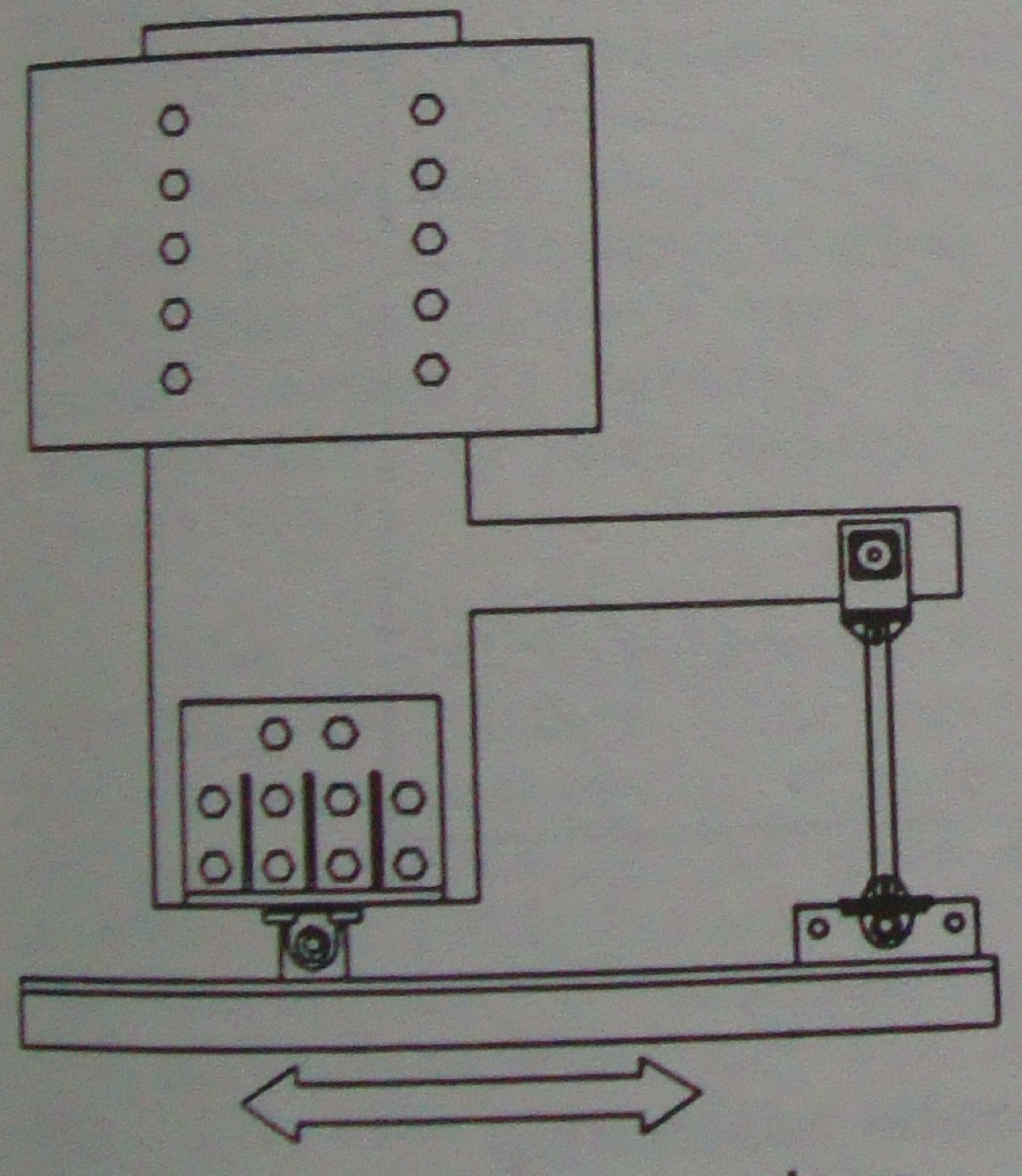

Figure 3. SDOF test specimen

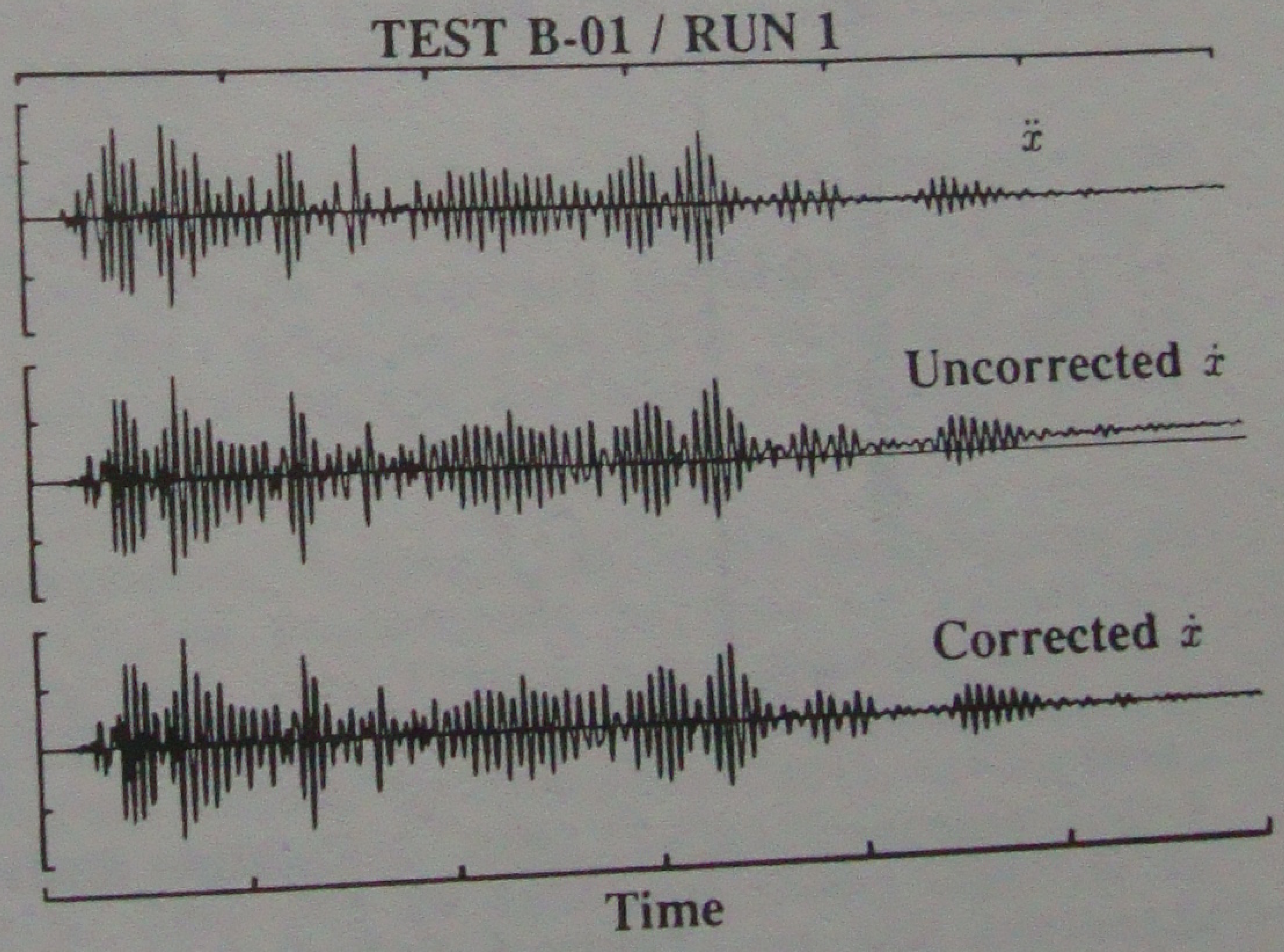

Figure 4. Relative velocity response signal

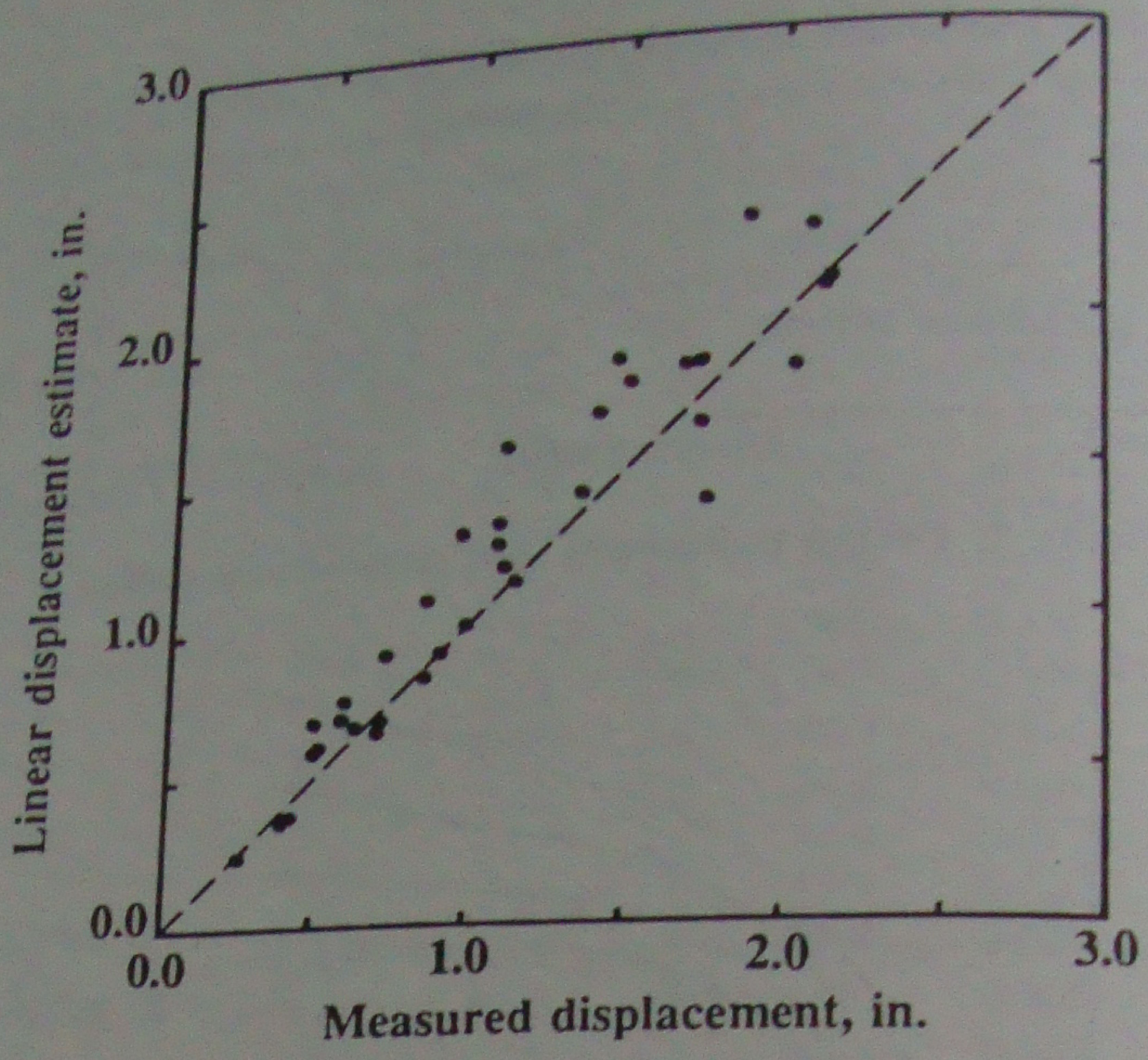

Figure 5. Evaluation of estimated displacements

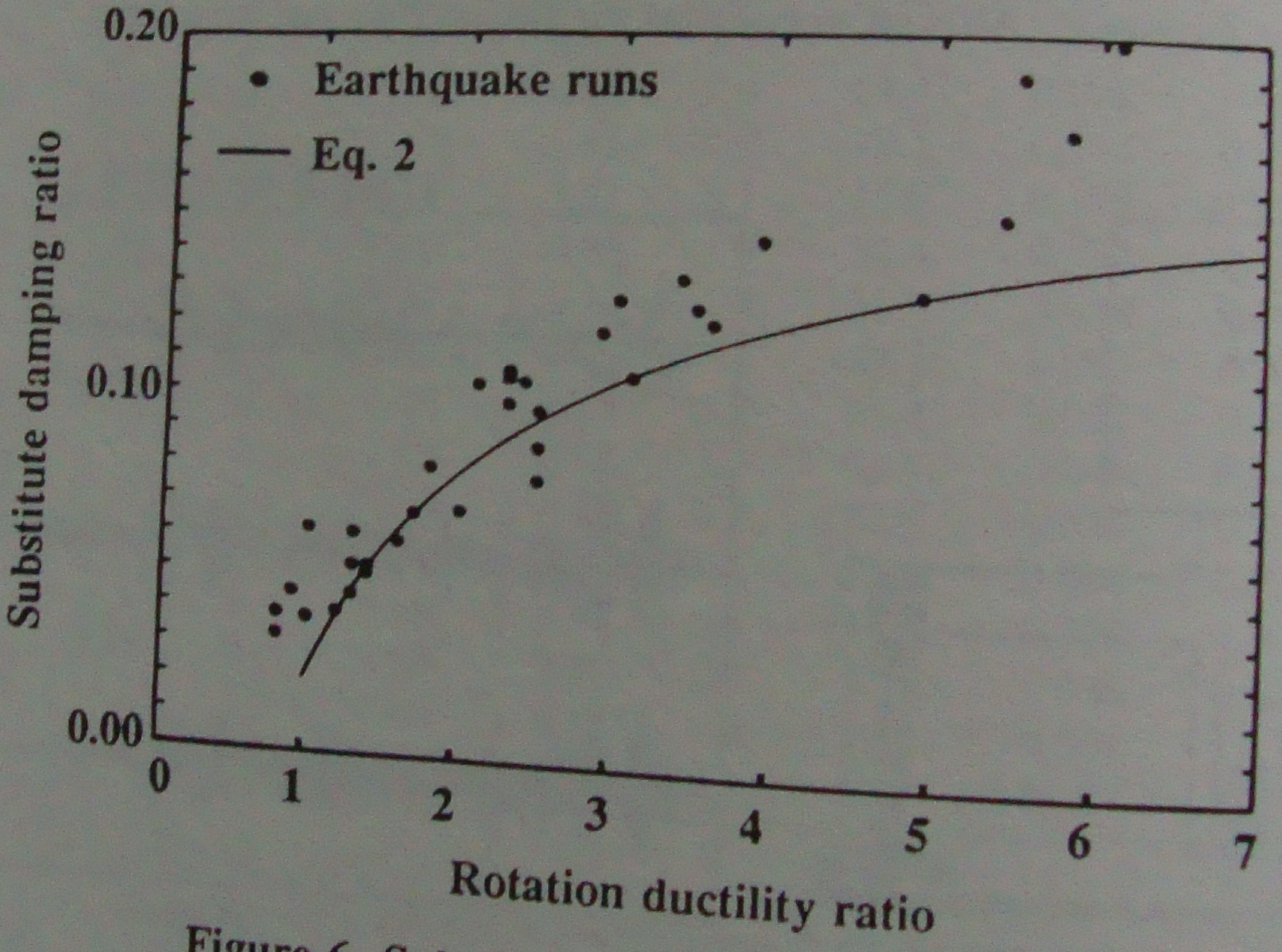

Figure 6. Substitute damping ratio relationship